Critical Plane Analysis of Fatigue Crack Nucleation Under Tension, Shear, and Compression

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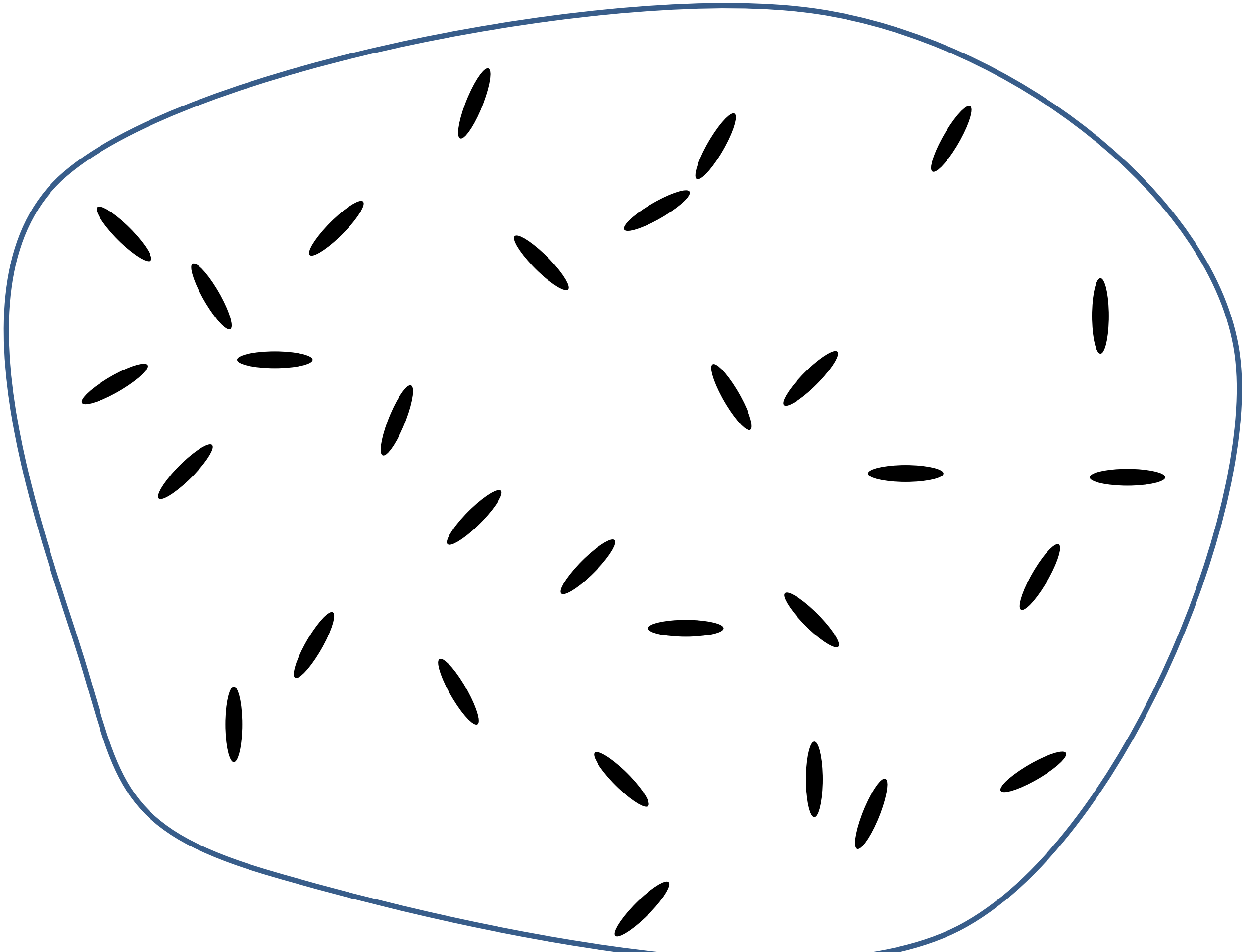
Agenda

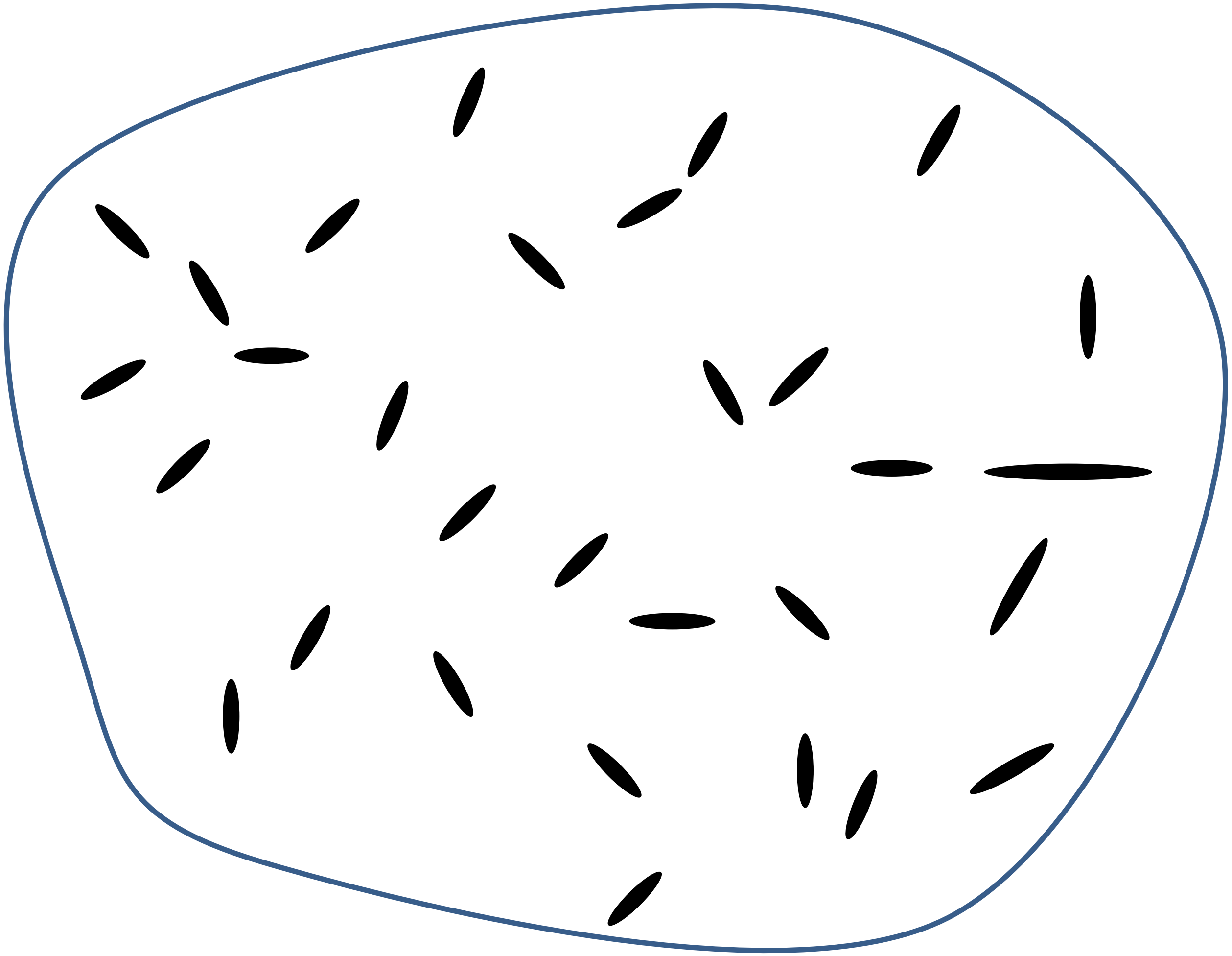
- Context
- Purpose
- Material Behavior
  - Stress-Strain
  - Fatigue Crack Growth
  - Microstructure / fatigue precursors
- Mechanics of Tension, Shear and Compression
- Critical plane analysis of crack nucleation
- Lessons learned

Body with crack precursors



N=0

Body with crack precursors



N=1000

Purpose

- How can we estimate fatigue behavior of a rubber part under practical loading scenarios?

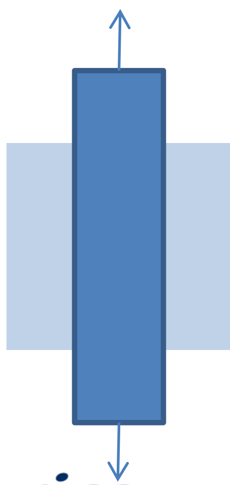
- Subject cases:
  - Tension
  - Shear
  - Compression

Loading Cases / Duty Cycle Definitions

$$\varepsilon = \varepsilon_a \cos(t) + \varepsilon_m \qquad \varepsilon_a = \varepsilon_m = 0.25, 0.375, 0.50, 0.75, 1.00$$

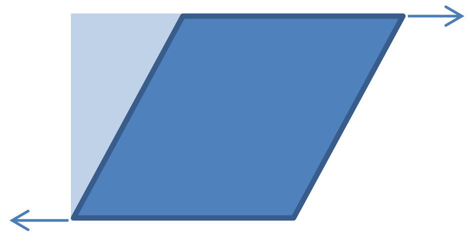
Simple Tension

$$F = \begin{bmatrix} 1+\varepsilon & 0 & 0 \\ 0 & (1+\varepsilon)^{-1/2} & 0 \\ 0 & 0 & (1+\varepsilon)^{-1/2} \end{bmatrix}$$



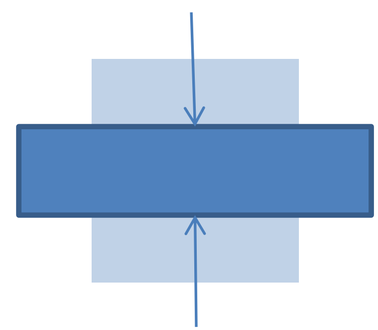
Simple Shear

$$F = \begin{bmatrix} 1 & (1+\varepsilon)-(1+\varepsilon)^{-1} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

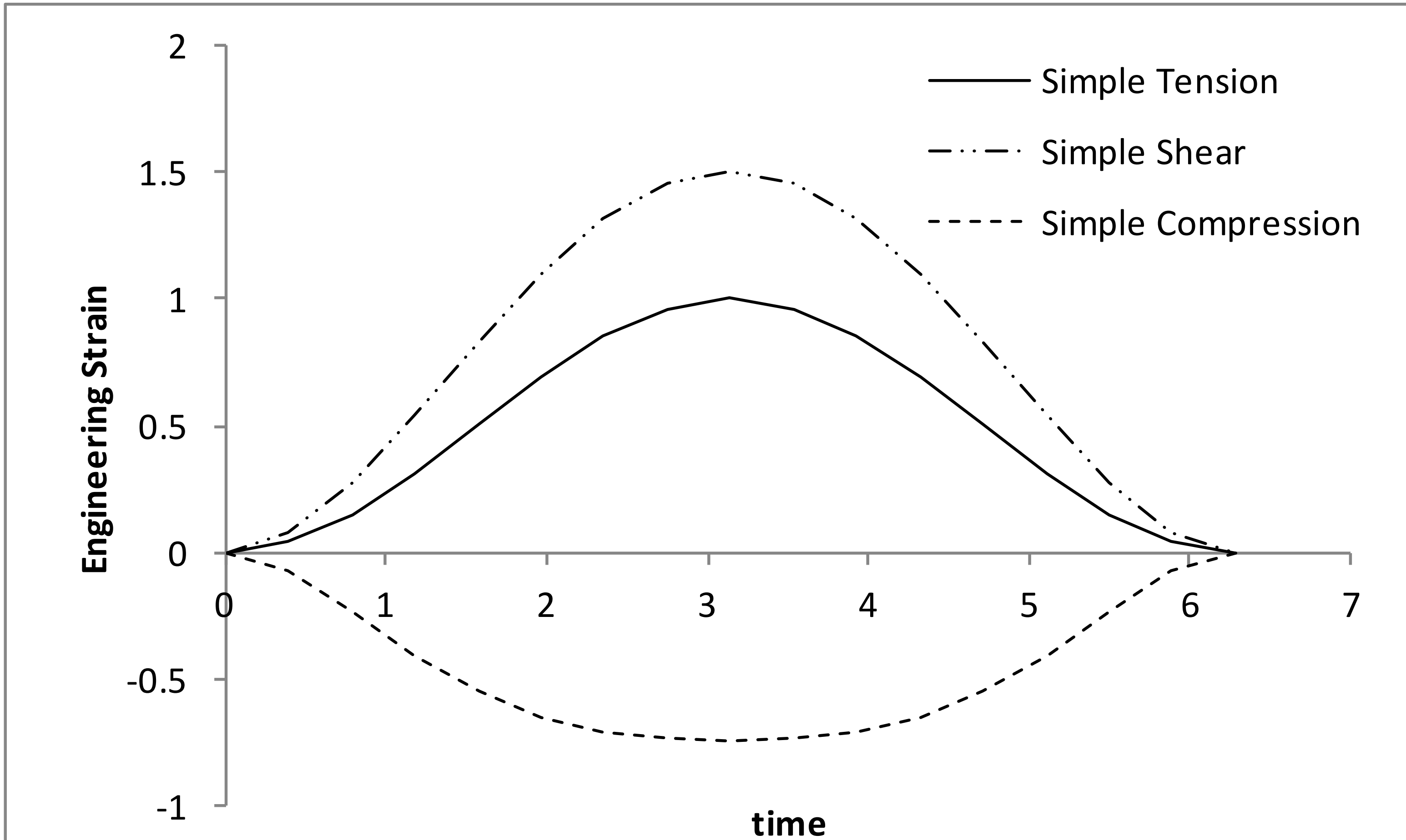


Simple Compression

$$F = \begin{bmatrix} \dfrac{1}{(1+\varepsilon)^2} & 0 & 0 \\ 0 & 1+\varepsilon & 0 \\ 0 & 0 & 1+\varepsilon \end{bmatrix}$$



Applied Duty Cycles in Terms of Nominal Strain



100% peak maximum principal strain

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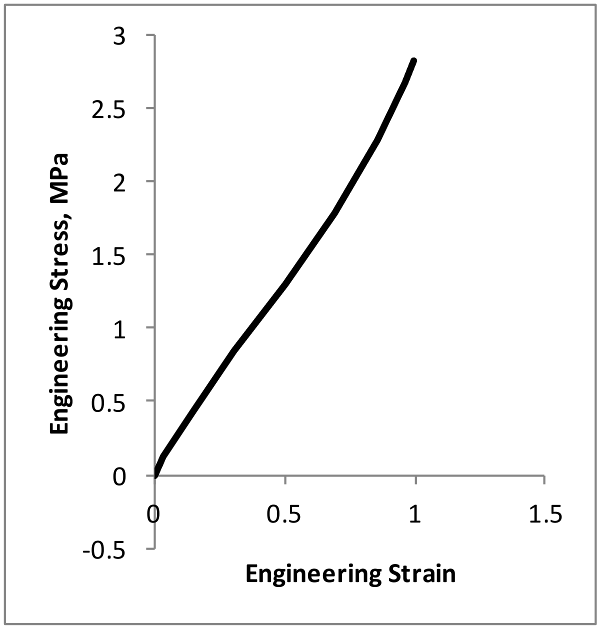
Ogden Hyperelastic Law

$$W = \sum_{i=1}^{n} \frac{2\mu_i}{\alpha_i^2} \left(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3 \right)$$

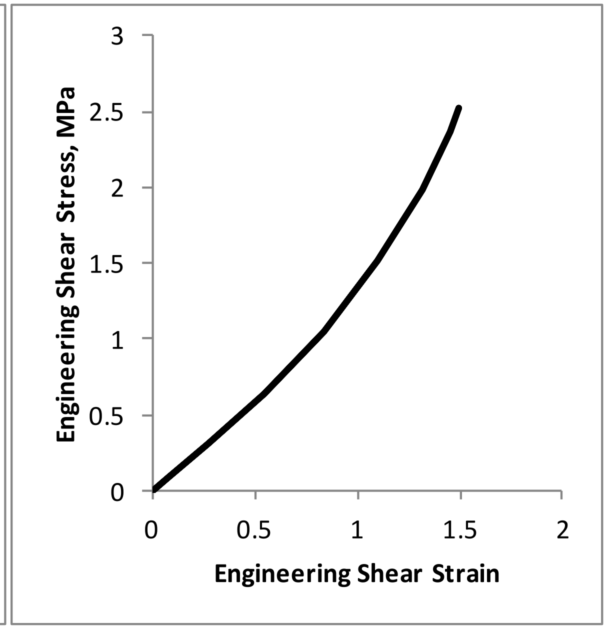
	i=1	i=2	i=3
μ, MPa	1.0	0.1	0.02
α	2.5	-3.5	8

Stress-Strain Behavior

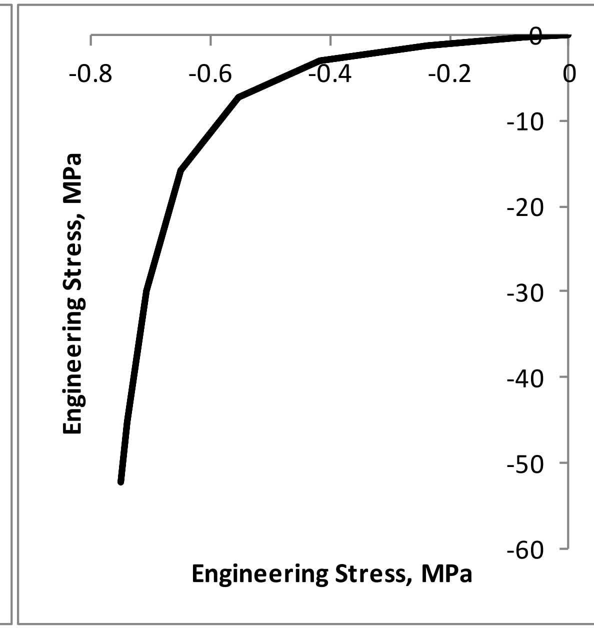
Simple Tension



Simple Shear



Simple Compression



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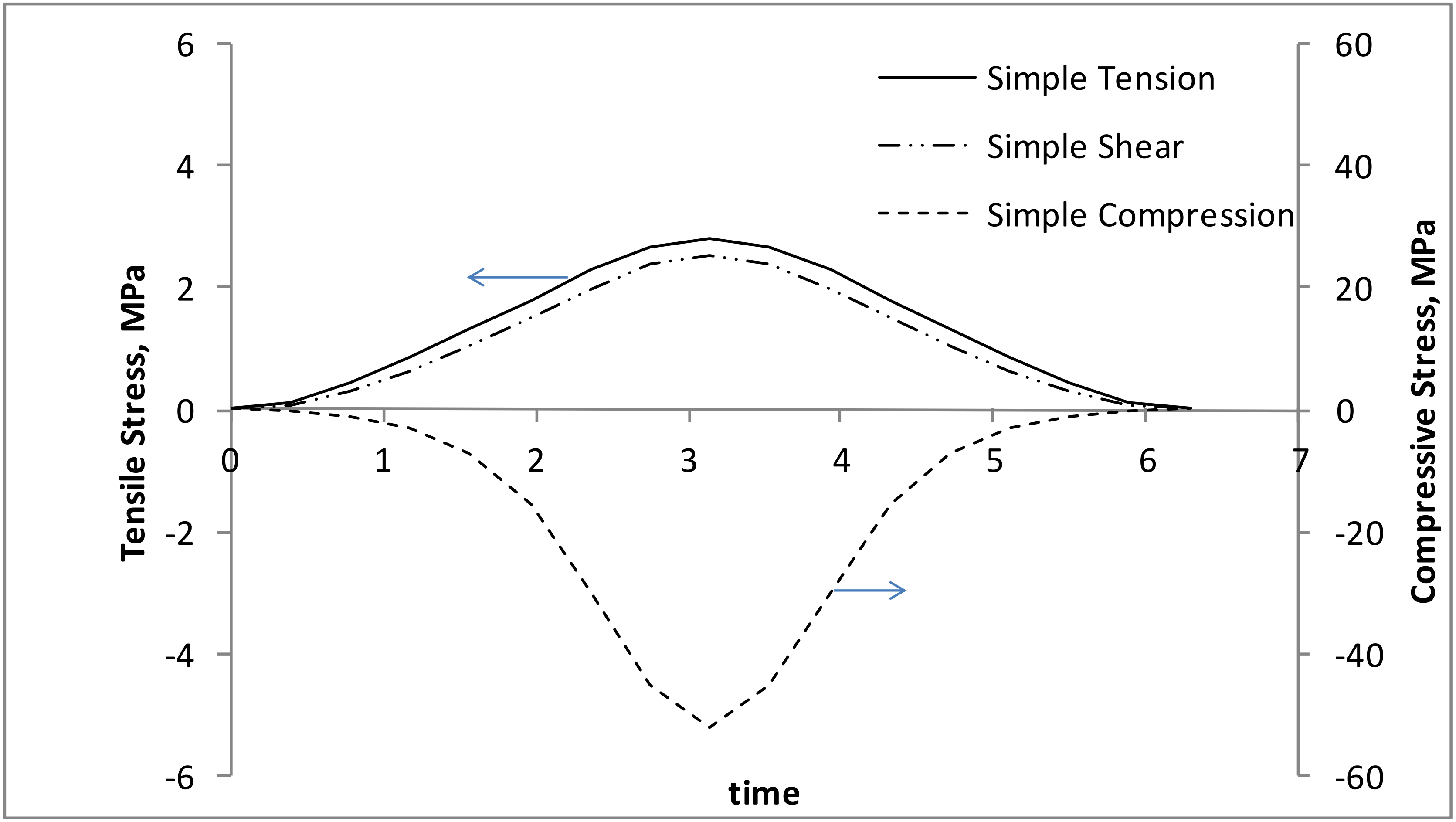
Applied Duty Cycles in Terms of Nominal Stress



100% peak maximum principal strain

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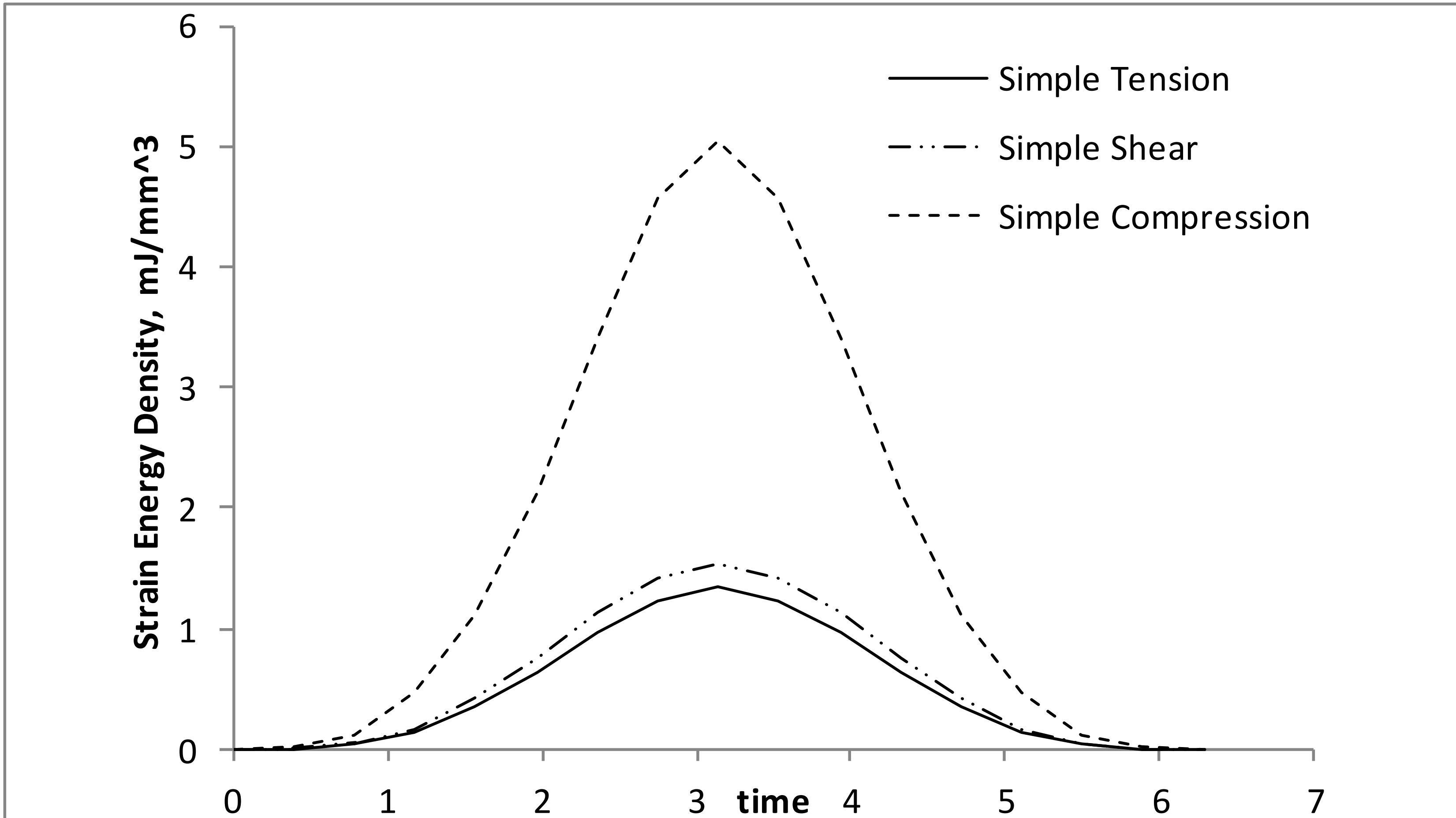
Applied Duty Cycles in Terms of Strain Energy Density



100% peak maximum principal strain

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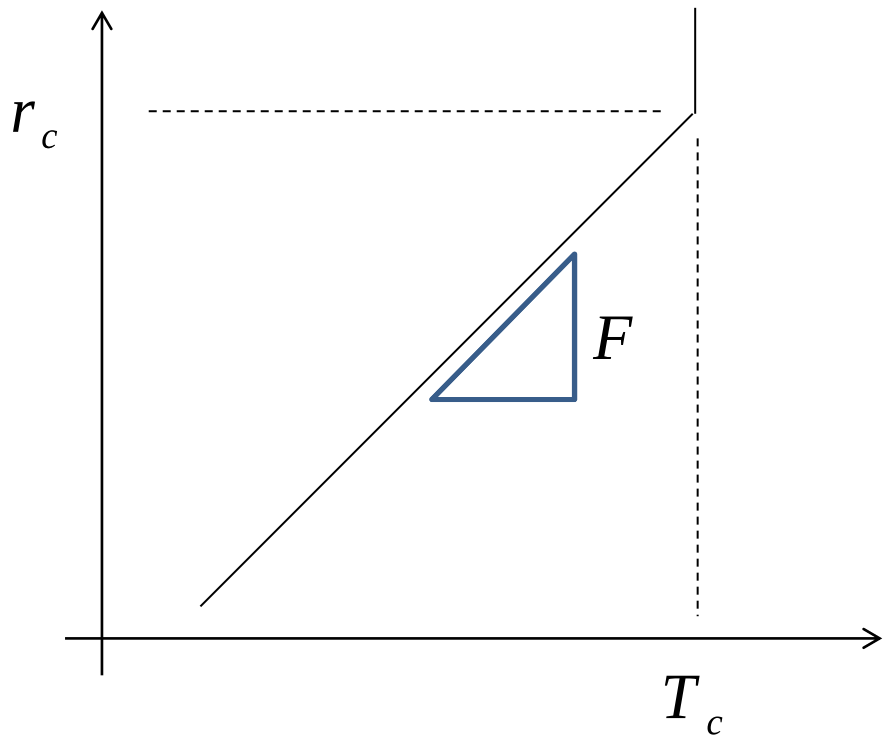
Fatigue Crack Growth Rate Law



$$r = r_c\left(\frac{T}{T_c}\right)^F$$

F = 2, T_c = 30 kJ/m^2, r_c = 1 x 10^{-3} mm/cyc

Flaw Size and Damage Law

$$N = \int_{c_0}^{c_f} \frac{dc}{r(T)}$$

$$c_0 = 50 \times 10^{-3}\,\mathrm{mm}$$

$$c_f = 1\,\mathrm{mm}$$

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Critical Plane Analysis

$$\varepsilon_{ij}(t)$$

$$\vec{r}(\theta,\phi)$$

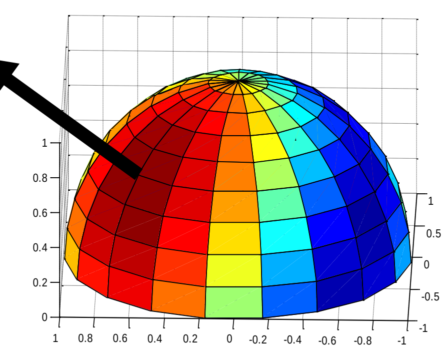
$$W_c(\theta,\phi,t) = \int_0^{\boldsymbol{\varepsilon}} \vec{r}^{\,T}\,\boldsymbol{\sigma}\,d\boldsymbol{\varepsilon}\,\vec{r}$$

$$\frac{dc}{dN} = f(T,R)$$

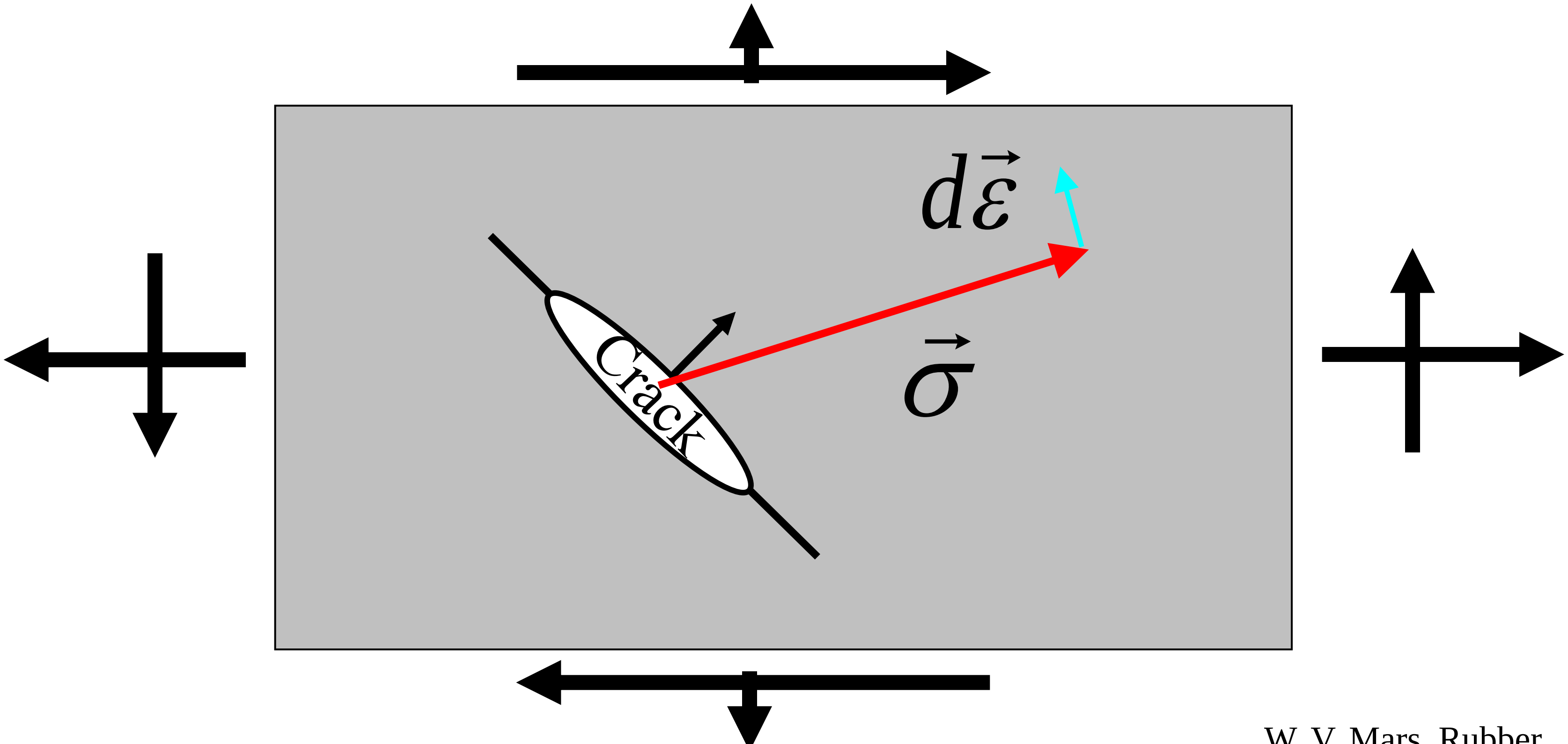
$$T_2(\theta,\phi,c) = 2\pi W_c c$$

$$N_f(\theta,\phi) = \int_{c_0}^{c_f} \frac{1}{f(T,R)}\,dc$$

$$\theta_{\min},\phi_{\min},N_{f,\min}$$



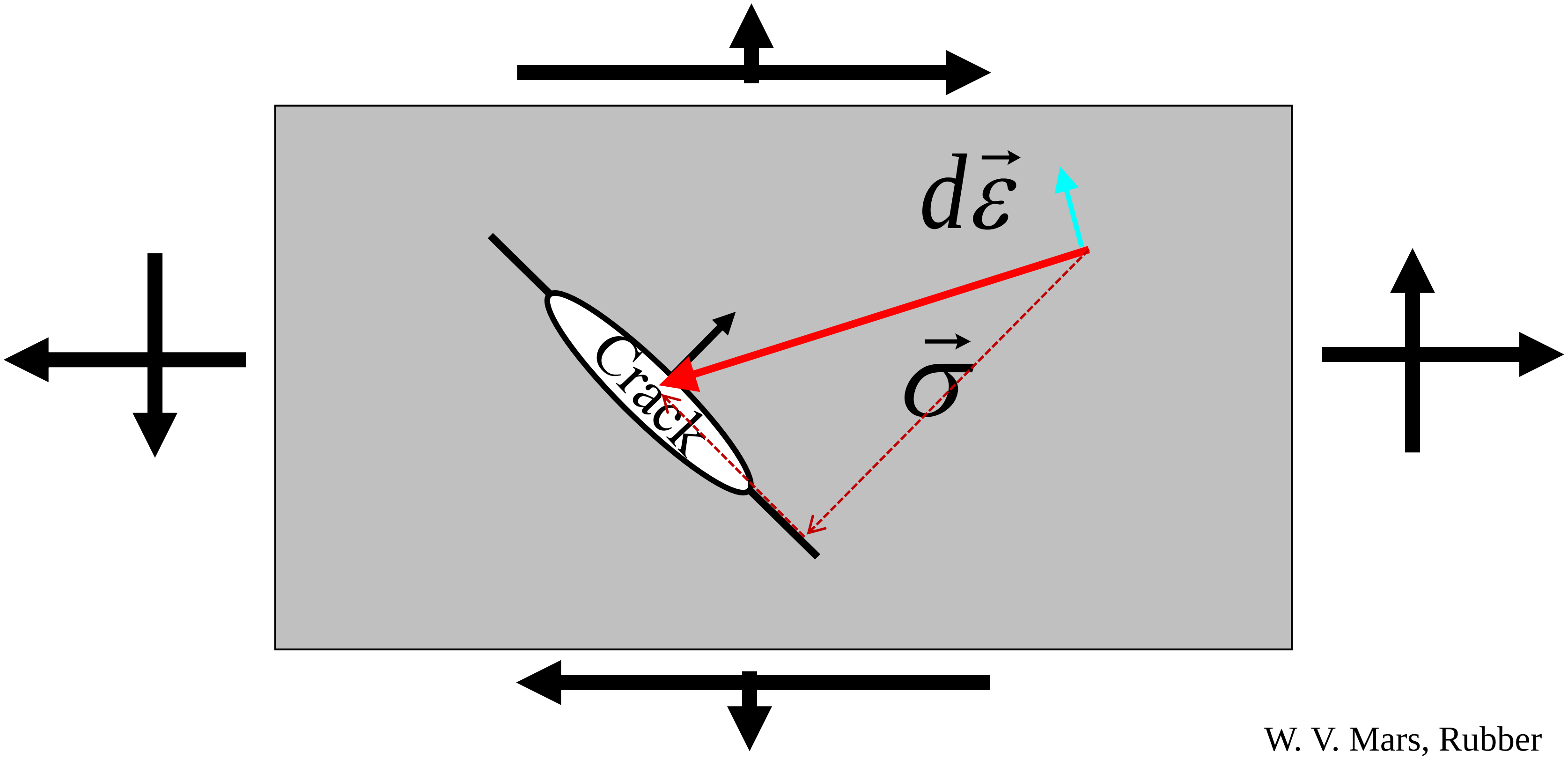
Cracking Energy Density



$$dW_c = \vec{\sigma} \cdot d\vec{\varepsilon}$$

W. V. Mars, Rubber Chemistry and Technology, Vol. 75, pp. 1-18, 2002.

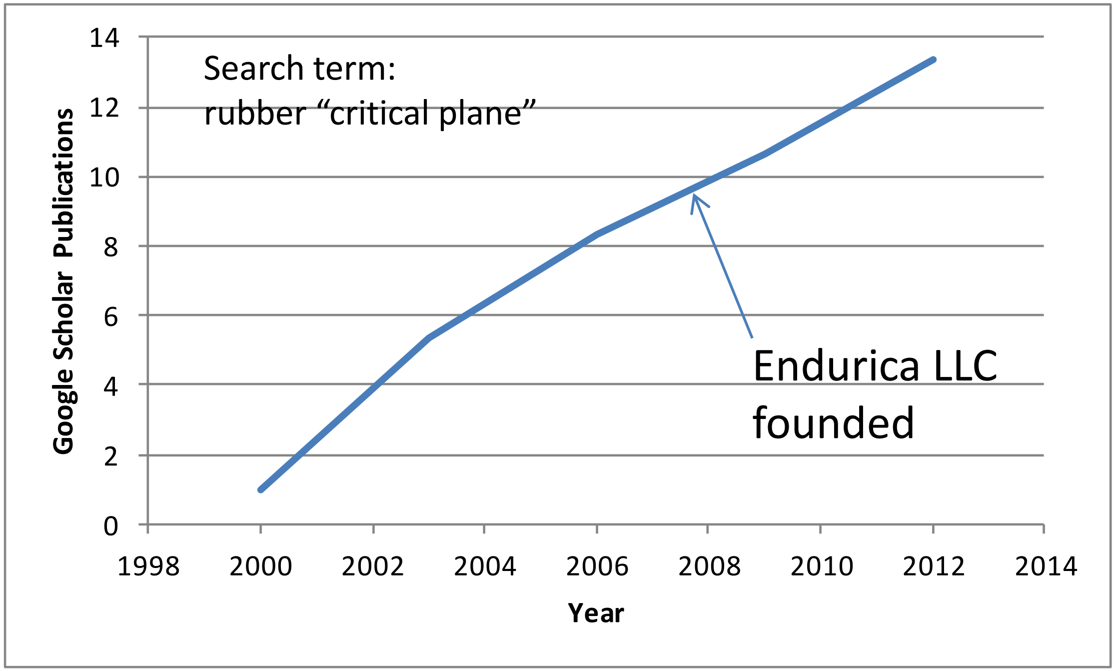
Crack Closure



$$dW_c = (\cancelto{0}{\vec{\sigma}_n} + \vec{\sigma}_t)\cdot d\vec{\varepsilon}$$

W. V. Mars, Rubber Chemistry and Technology, Vol. 75, pp. 1-18, 2002.

Critical Plane Analysis



~30% growth per year in rate of publications

Endurica fatigue solver technology is available commercially as

fe-safe/Rubber™

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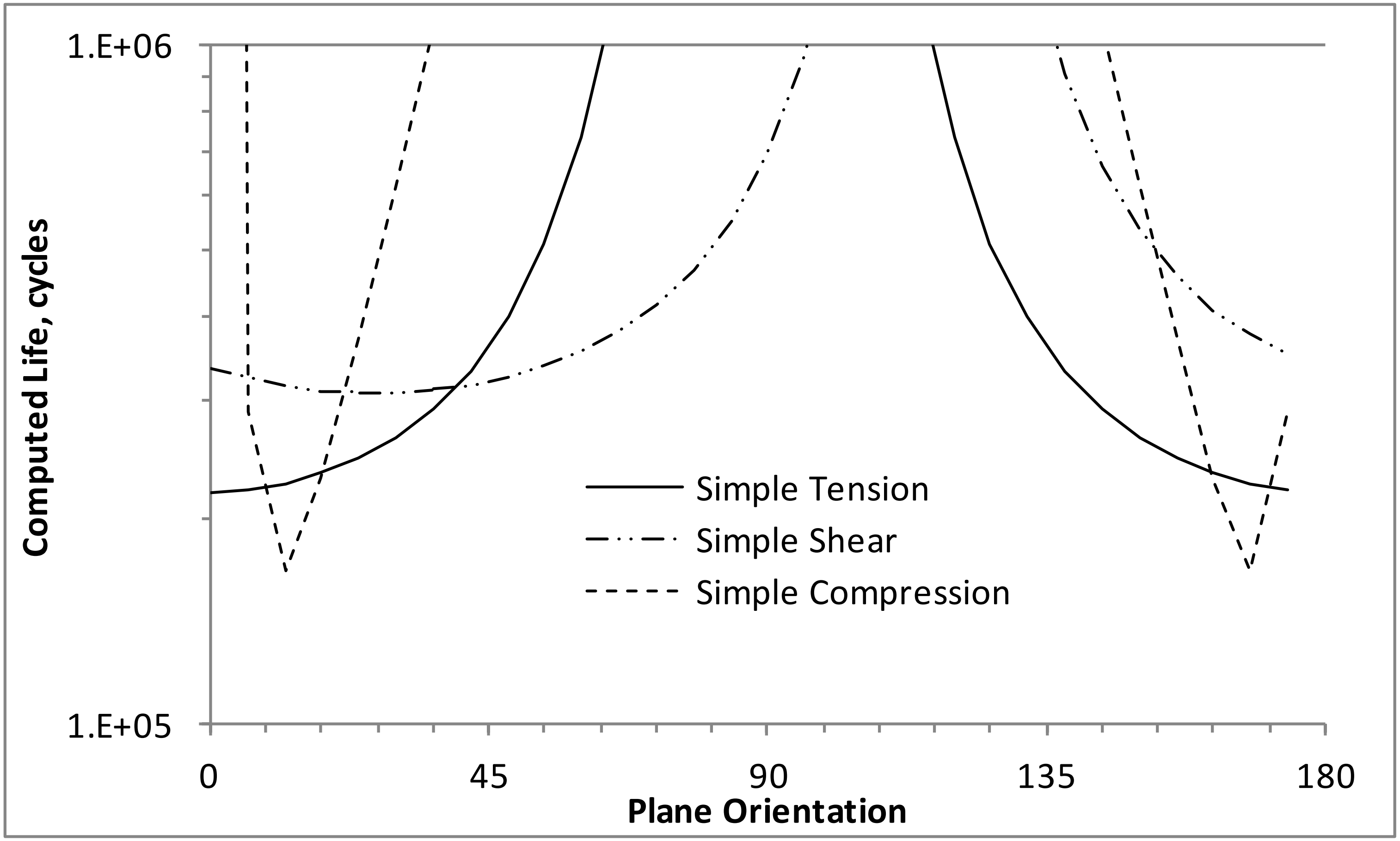
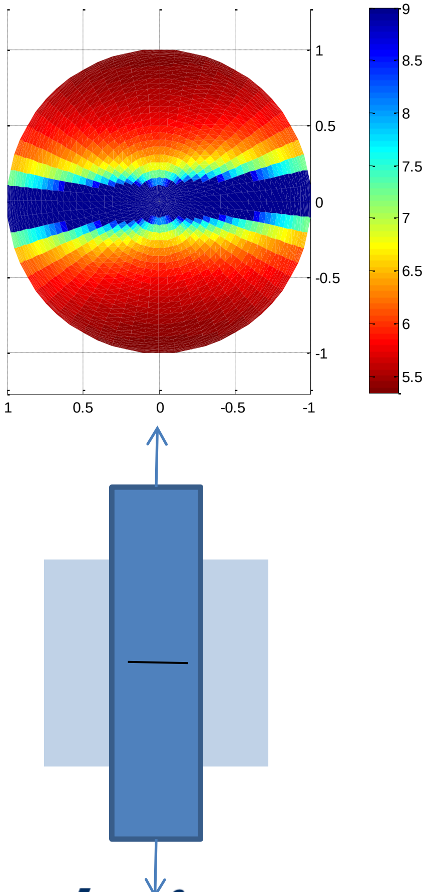
Critical Plane Analysis

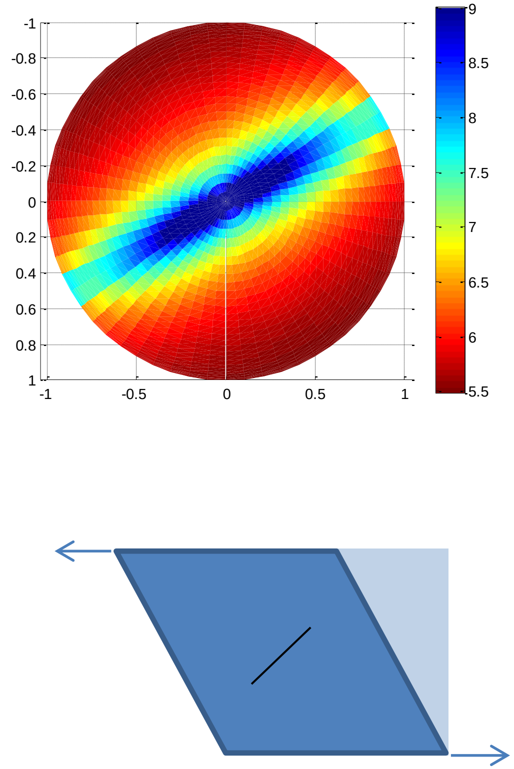


Damage Sphere

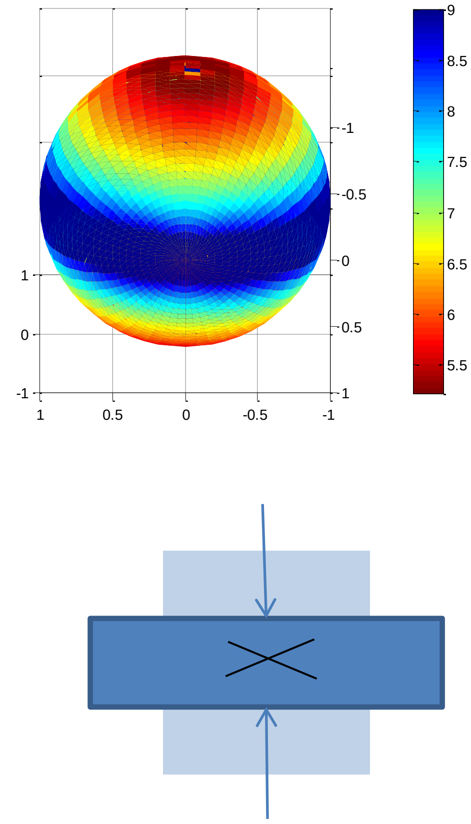
Simple Tension



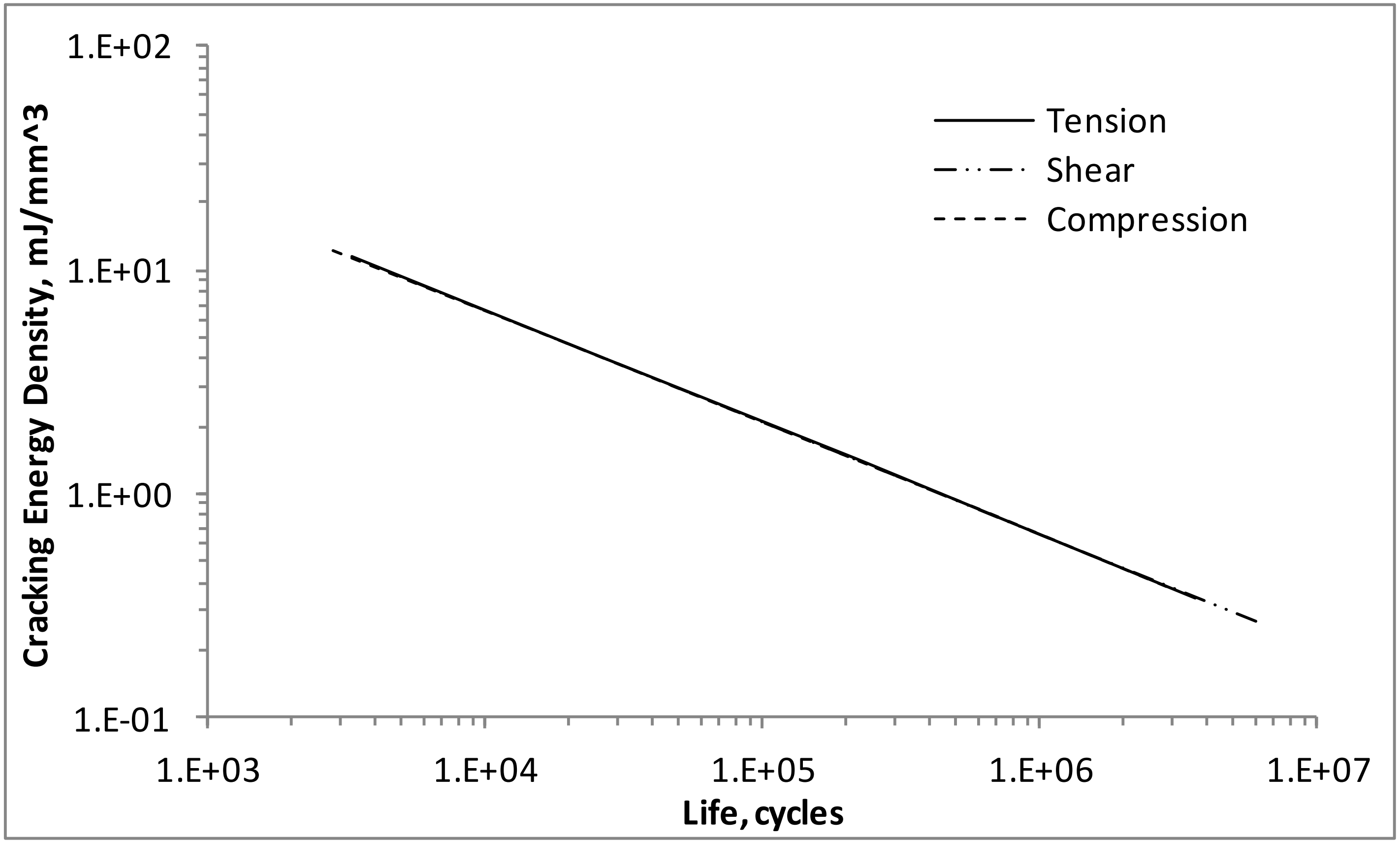
Simple Shear



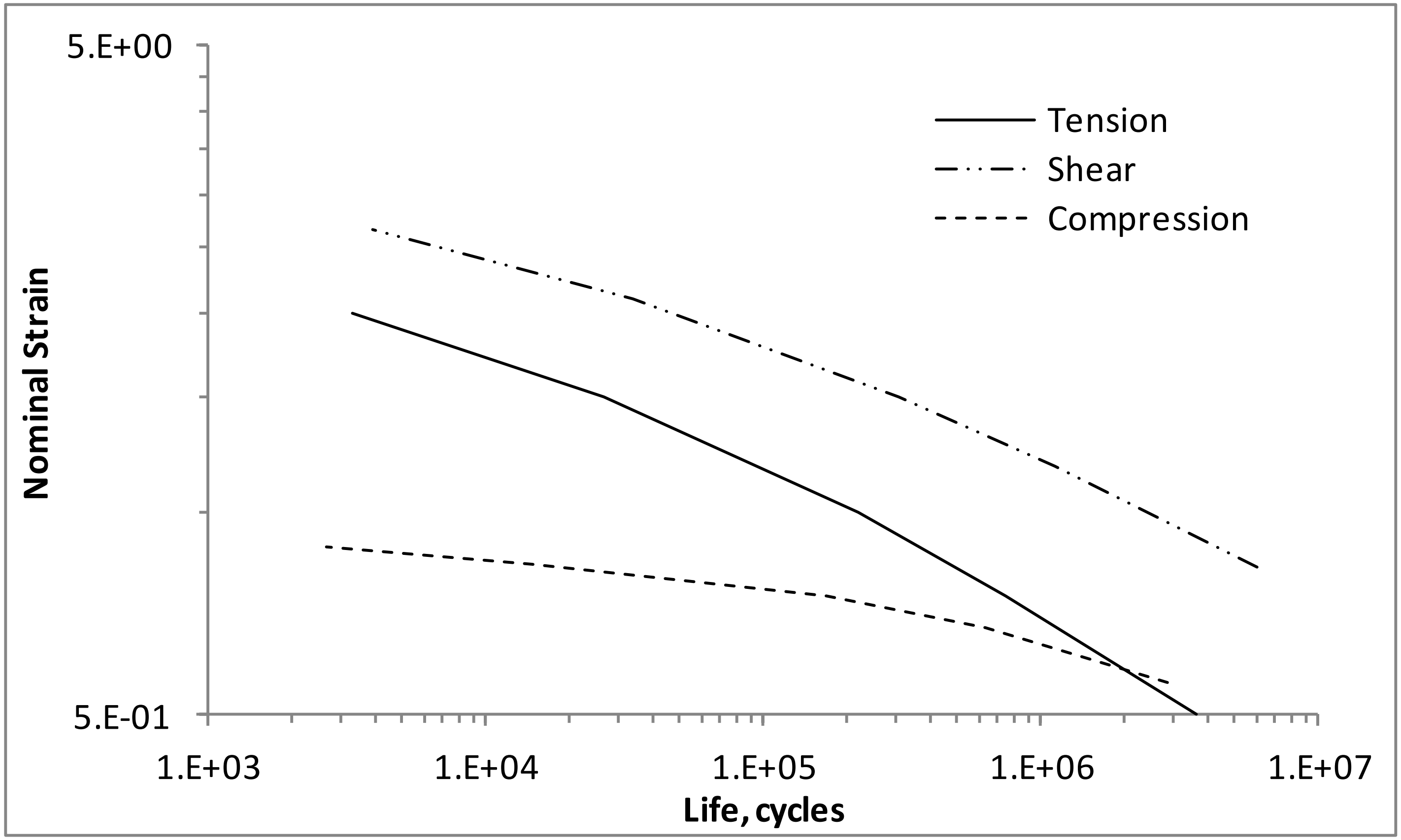
Simple Compression



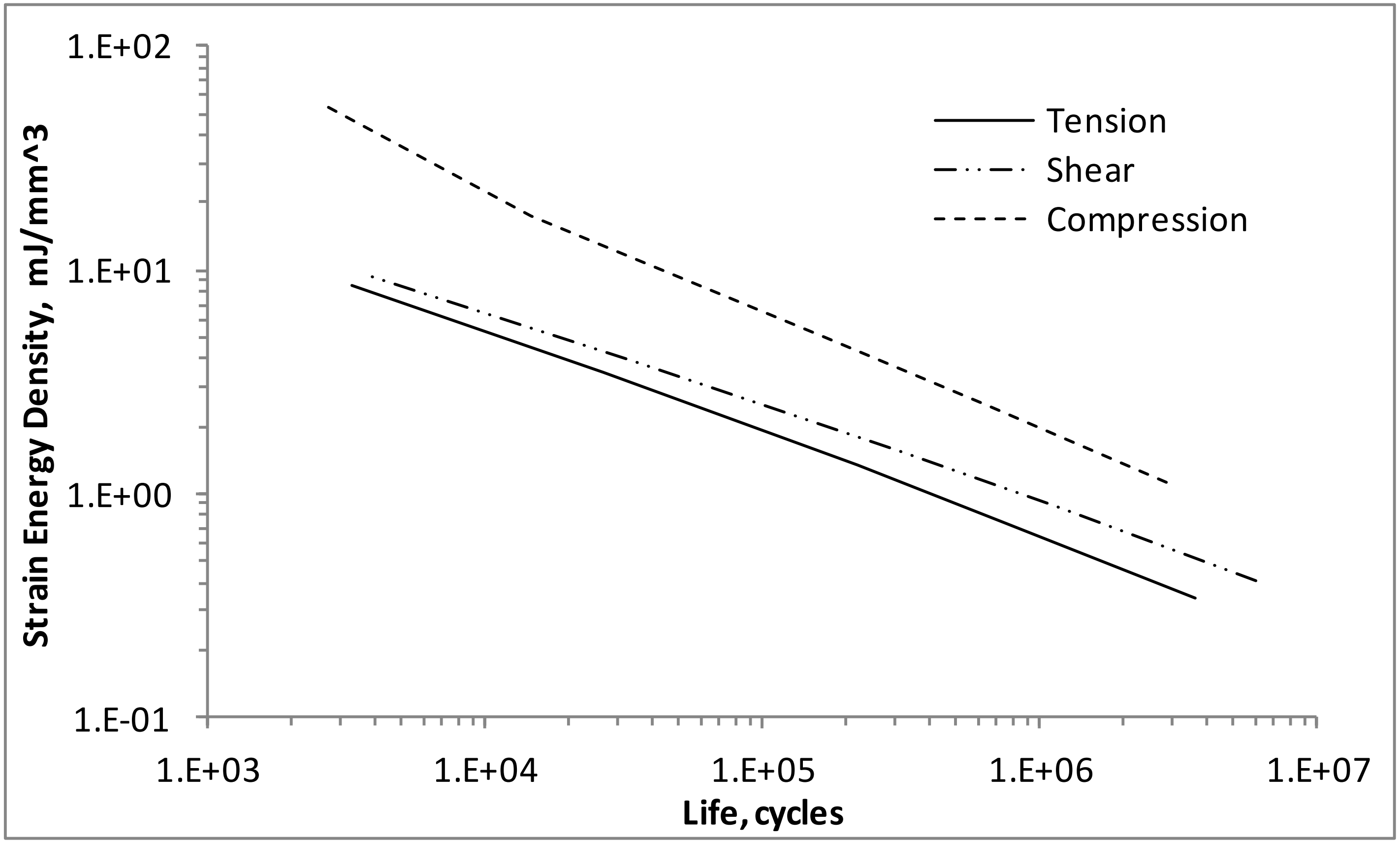
Computed CED-Life



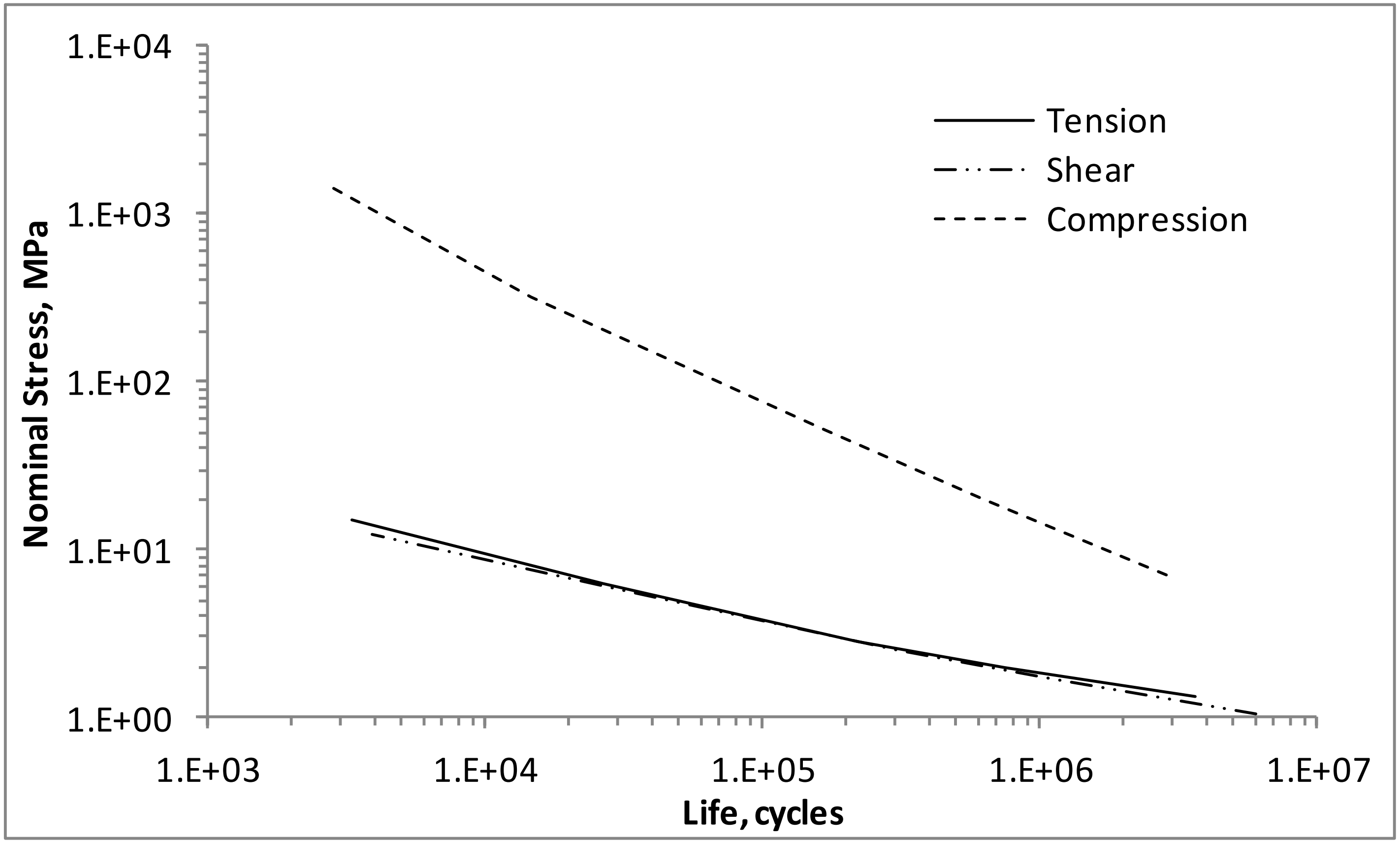
Computed Strain-Life Curve



Computed SED-Life



Computed Stress-Life

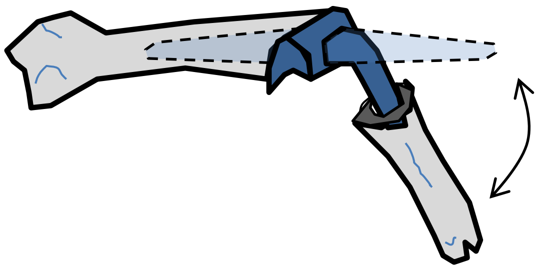


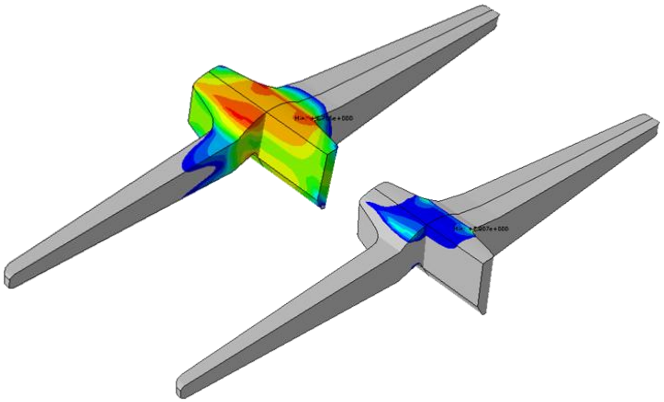
Conclusion

- Critical plane analysis = considering all possible failure planes
  - Identify the worst plane
  - Estimate local experience of crack precursor
- Simple loads with tensile max prin stress = crack orientation normal to prin stress
- Critical plane analysis needed whenever
  - No principal stress is tensile (ie crack closure)
  - Nonrelaxing loads
  - Loads with variable principal directions
- Standard parameters governing fatigue behavior
  - Stress-strain law
  - FCG law
  - Precursor size
- X-N curves differ depending on mode of deformation, but can be calculated using numerical analysis

The Future

- Duty cycles modeled via FEA
- Variable amplitude, variable mode loading
- Characterization and modeling of fatigue for product development





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Abstract

- When fatigue cracks develop from microscopic precursors in rubber, they tend to do so on specific planes. The orientation and loading experiences of such planes under the action of a given mechanical duty cycle directly govern the rate at which associated cracks develop, and ultimately the fatigue life. Here we demonstrate the consequences of this principle for three loading scenarios that are common in elastomer components: simple tension, simple shear, and simple compression. For each case, we first identify a prospective failure plane, we consider the associated local mechanical duty cycle, and we estimate the rate of damage accumulation for the prospective plane. After considering all possible planes, we then identify the most critical plane(s) as the one (or several) plane(s) that has(ve) the maximum rate of damage accumulation. The analysis illuminates how fatigue behavior can be expected to differ between tension, shear, and compression, and how these differences derive from the same fundamental material behaviors.

- Keywords: Fatigue, Failure, Mechanics, Multiaxial, Critical Plane Analysis