Where do the "Pure" and "Shear" come from in the Pure Shear test?

Problem Definition

The Pure Shear test piece is widely applied to characterize the stress-strain, strength, and fatigue properties of soft materials. It consists of a thin rectangular sheet that is gripped or bonded along its long edges to prevent lateral contraction (in the 2 direction) as it is extended in the direction of its short edges (the 1 direction). The sheet may freely contract in its thickness dimension (the 3 direction). In order to achieve sufficient constraint and homogeneity, the width of the specimen should be greater than the height by at least 8 times. The test piece is also known by the names Constrained Tension and Planar Tension. The test piece is shown schematically in Figure 1.

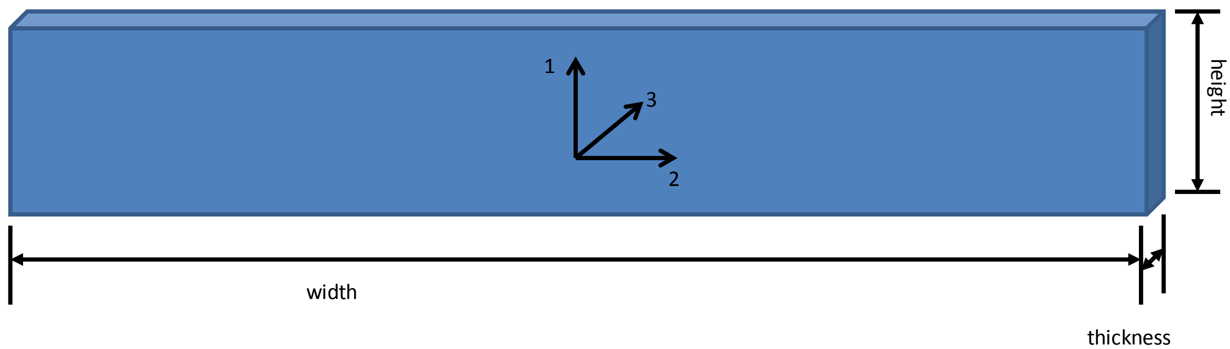

Figure 1. Pure shear test piece dimensions. Strain is applied in the 1 direction while strain in the 2 direction is prevented.

The term "shear" refers to a deformation in which parallel planes remain parallel and are displaced laterally relative to each other, skewing lines that were originally perpendicular to the displacement direction. Take the Simple Shear test piece, for example, shown in Figure 2. It is gripped or bonded along its long edges. The deformation is applied by displacing the top grip parallel to the bottom grip.

Figure 2. Simple shear test piece. Strain is applied via the lateral displacement of the grips, resulting in skewing of lines originally perpendicular to the lateral displacement.

Many people puzzle over the nomenclature of the pure shear test. They rightly point out that 1) the Pure Shear test piece is loaded in tension by extending the specimen in the axial direction, and 2) a shearing deformation, by definition, involves the lateral motion of parallel planes. They wonder where is the "shear"? and what does it mean to say that the shear is "pure"?

Analysis

Let's look into this a bit further. First, let's describe the state of deformation in a homogeneously strained Pure Shear test piece. The deformation gradient **F** is written in terms of the axial stretch λ. We have assumed incompressibility here.

$$\mathbf{F} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda^{-1} \end{bmatrix}$$

We then compute the velocity gradient tensor **L** as

$$\mathbf{L} = \dot{\mathbf{F}}\mathbf{F}^{-1} = \mathbf{D} + \mathbf{W} = \frac{\dot{\lambda}}{\lambda}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Note that **D** is the symmetric rate of deformation tensor, here identical with **L**, because of **L**'s symmetry. This also implies that the anti-symmetric spin tensor **W** is zero.

Finally, we construct Mohr's circle for the rate of deformation tensor. This is shown in Figure 3.

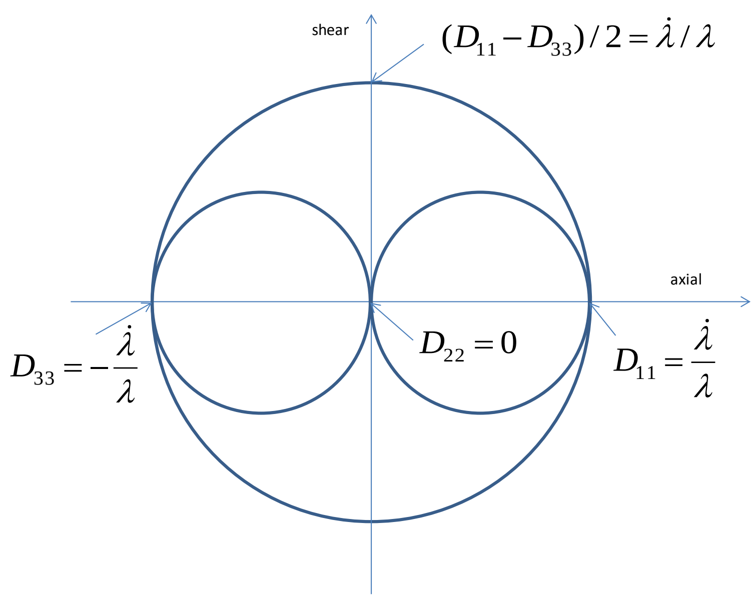

Figure 3. Mohr's circle on the rate of deformation tensor for the Pure Shear test piece.

We see that the Mohr's circle connecting the major and minor principal deformation rates is centered at zero. Thus, the value of the maximum shear deformation rate is equal to the major principal deformation rate, and on the plane of the maximum shearing rate, the associated axial deformation rate is zero. This is the first sense in which the subject test piece is producing a "Pure Shear".

This analysis also shows us where to look to see the shearing action in the applied deformation. To see this, we call attention to the material element that is rotated 45 degrees from the 1 axis. As shown in Figure 4, as axial deformation is applied to the specimen, this particular material element maintains parallel planes that experience lateral relative displacement. We also see that there is no rigid body rotation of the overall material element as it deforms, only the shearing motion induced by the axial extension of the grips. This is the second sense in which the test piece is producing a "Pure Shear" – it is pure in the sense that there is no rigid body rotation of the resulting deformation state. This may be contrasted with the simple shear test piece, in which the shearing motion can occur only in association with a rigid body rotation of the material.

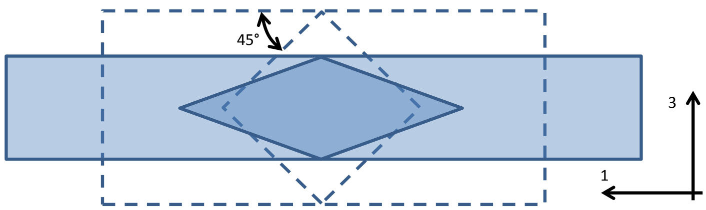

Figure 4. End view (looking down the 2 direction) of the pure shear test piece in the original (dotted lines) and deformed (solid lines) states. The material element oriented at 45° to the 1 direction distorts in shear as axial loading is applied in the 1 direction.

Conclusion

The deformation state occurring in the Pure Shear test piece is so named because

1) The axial rate of deformation on the plane of maximum shearing is identically zero at all times
2) There is no rigid body rotation of the material in the pure shear test

Challenge

Starting from the deformation gradient for the simple shear case, compute the velocity gradient **L**, the rate of deformation tensor **D**, and the rate of spin tensor **W**, and construct Mohr's circle for the rate of deformation tensor. The deformation gradient **F** in simple shear is

$$\mathbf{F} = \begin{bmatrix} 1 & \gamma & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In what sense is simple shear similar to pure shear? In what sense does it differ?