How Does Pure Shear Compare to Simple Shear?

In a prior analysis, we considered the nomenclature of the Pure Shear test piece, and showed that the associated deformation state is so named for two reasons: 1) on the plane with maximum shear rate, the associated axial component of the rate of deformation tensor is identically zero at all times, and 2) there is no rigid body rotation of the material.

The question was raised, however, of how the Pure Shear deformation compares to the Simple Shear deformation. The Pure Shear test piece is shown in Figure 1, and the Simple Shear test piece is shown in Figure 2.

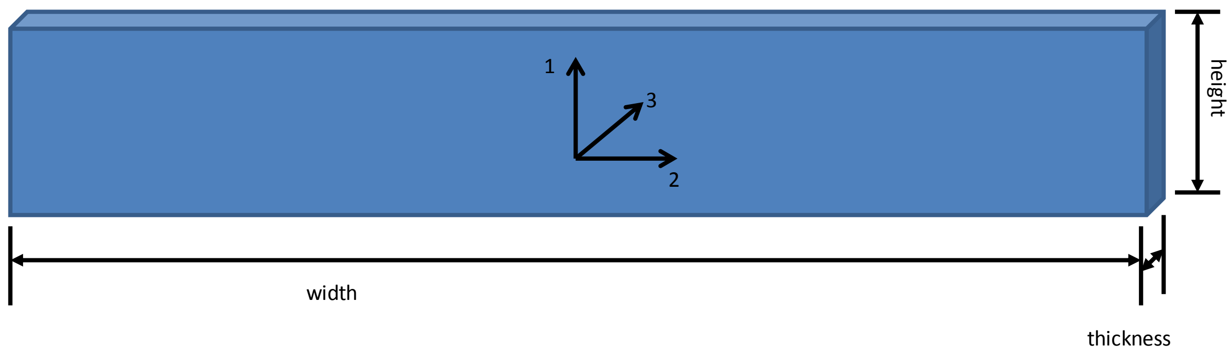

Figure 1. Pure shear test piece coordinate system. Strain is applied in the 1 direction.

Figure 2. Simple shear test piece.

Analysis

We first recall the analysis results for the Pure Shear test piece. The deformation gradient **F** was written in terms of the axial stretch λ. We have assumed incompressibility.

$$\mathbf{F} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda^{-1} \end{bmatrix}$$

We found that the rate of deformation tensor **D** is

$$\mathbf{D} = \frac{\dot{\lambda}}{\lambda} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

And that the anti-symmetric spin tensor **W** is zero.

We constructed Mohr's circle for the rate of deformation tensor, as shown in Figure 3. Because Mohr's circle is centered on zero, we conclude that the planes with maximum shearing (i.e. the +/- 45 degree planes cutting the 1-3 plane) have an associated axial rate of deformation of zero. The shearing is "pure" in this sense.

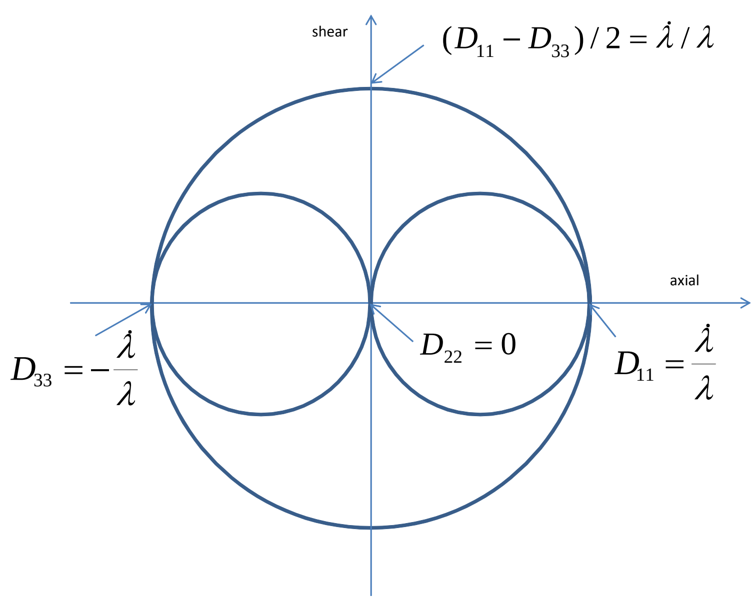

Figure 3. Mohr's circle on the rate of deformation tensor for the Pure Shear test piece.

The deformation gradient **F** in simple shear is

$$\mathbf{F} = \begin{bmatrix} 1 & \gamma & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For simple shear, we compute the velocity gradient **L** as

$$\mathbf{L} = \dot{\mathbf{F}}\mathbf{F}^{-1} = \mathbf{D} + \mathbf{W} = \dot{\gamma}\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \frac{\dot{\gamma}}{2}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{\dot{\gamma}}{2}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

In this case, the spin tensor **W** is non zero, implying a rigid body motion of the material. The eigenvalues of **D** are $D_{11} = \frac{\dot{\gamma}}{2}, D_{22} = -\frac{\dot{\gamma}}{2}, D_{33} = 0$, so that we can draw Mohr's circle as shown in Figure 4.

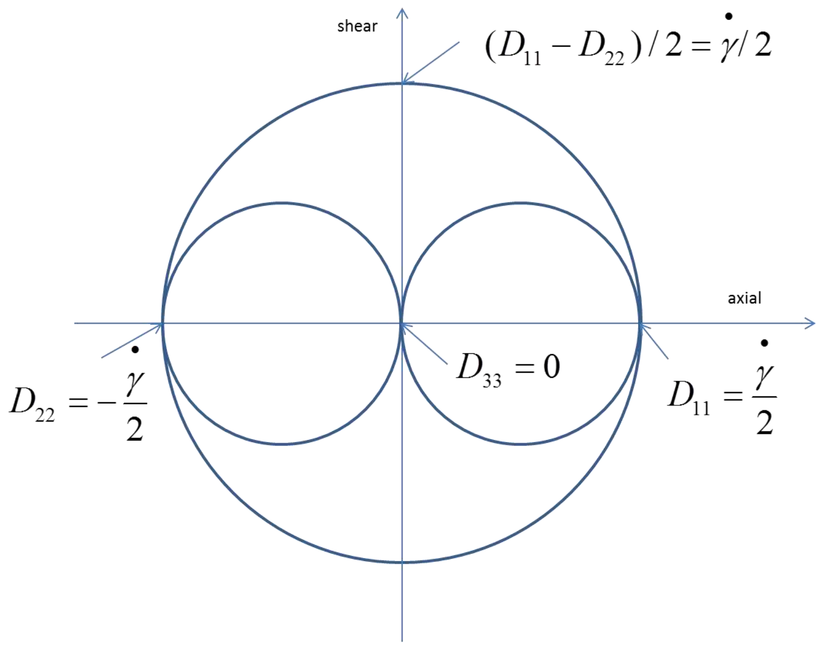

Figure 4. Mohr's circle on the rate of deformation tensor for the Simple Shear test piece.

Conclusion

We thus identify that the rate of deformation in the Pure Shear experiment $\frac{\dot{\lambda}}{\lambda}$ is identical to one half of the rate of shearing $\dot{\gamma}$ in the Simple Shear experiment. Also, we see that the deformation achieved in the two experiments differs fundamentally only in this one respect: that while in the pure shear test, no rigid body rotation arises as the material element is extended, and while in the simple shear test, shearing is always accompanied by a rigid rotation of the material element at the rate $\dot{\gamma}/2$.